Mesoscopic gap fluctuations in an unconventional superconductor

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We study mesoscopic disorder fluctuations in an anisotropic gap superconductor, which lead to the spatial variations of the local pairing temperature and formation of superconducting islands above the mean-field transition. We derive the probability distribution function of the pairing temperatures and superconducting gaps. It is shown that above the mean-field transition, a disordered BCS superconductor with an unusual pairing symmetry is described by a network of superconducting islands and metallic regions with a strongly suppressed density of states due to superconducting fluctuations. We argue that the phenomena associated with mesoscopic disorder fluctuations may also be relevant to the high-temperature superconductors, in particular, to recent STM experiments, where gap inhomogeneities have been explicitly observed. It is suggested that the gap fluctuations in the pseudogap phase should be directly related to the corresponding fluctuations of the pairing temperature.

Understanding the phase diagram and the properties of the high-temperature and other unconventional superconductors has been among the most complex problems of modern condensed matter physics. Most current theoretical approaches to the problem concentrate on strong correlation physics and usually assume that the effects of disorder are unimportant. However, there exist a number of recent experimental works, in particular STM studies of the high-T_c cuprates,[1,2,3,4,5,6,7] which provide a tentative indication that at least in some materials disorder plays an important role in the local formation of the superconducting gap. In particular, Gomes et al.[2] have studied the local development of the gap as a function of temperature in $Bi_2Sr_2CaCu_2O_{8+\delta}$ above the superconducting transition and up to a pseudogap temperature, where the gap inhomogeneities cease to exist. An important result of this experiment is that the real-space gap map observed was static and reproducible. This strongly suggests that the inhomogeneous gap formation is unlikely to be a phase-separation or superconducting fluctuation effect, but is due to some kind of disorder in the system.

Motivated by these experiments, we theoretically consider a disordered superconductor with an unusual pairing symmetry (e.g., a d-wave superconductor) and study mesoscopic variations of the local pairing temperature. We point out that the existence of the Griffiths-type[8,9,10] phase in the superconducting phase diagram is specific to an anisotropic gap superconductor and should not occur in the conventional s-wave systems (due to Anderson theorem), unless they are extremely dirty or time-reversal symmetry is broken.[11,12,13] An important observation is that if the pairing gap is anisotropic, the Anderson theorem breaks down and the superconducting pairing temperature, T_p, is suppressed by disorder even if time-reversal symmetry is preserved (here and below we make a distinction between the pairing temperature, T_p, and the superconducting transition temperature, T_c, although in the framework of the weak-coupling BCS theory they are essentially the same). The impurities are positioned randomly in space and their density is a random variable. Thus, there always exist regions where the distribution of impurities is such that the local pairing temperature, $T_p(\mathbf{r})$, is larger than the system-wide average value, $\langle T_p \rangle$, and the experimental temperature, T.[14] These regions form islands with a well-

defined gap, which exist on the background of a metal, if $T_p(\mathbf{r}) > T > \langle T_p \rangle$. The width of the "mesoscopic fluctuation" region certainly depends on the strength of disorder and the only parameter, which may enter this dependence, is the dimensionless conductance. This defines a narrow window where the disorder-induced Griffiths phase co-exists with strong superconducting fluctuations. Therefore, the picture of impurity induced inhomogeneities in an anisotropic gap BCS superconductor is that of superconducting islands and metallic regions with strongly suppressed density of states.

We start with the following Hamiltonian with a built-in l-wave pairing ($l > 0$):

$$\hat{\mathcal{H}} = \int d^2\mathbf{r} \left\{ \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m} - \mu + U(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}) - \lambda_l \hat{b}^\dagger(\mathbf{r}) \hat{b}(\mathbf{r}) \right\}, \quad (1)$$

where $U(\mathbf{r})$ is a disorder potential, λ_l is the l-wave interaction constant, and $b(\mathbf{r})$ corresponds to an "l-wave Cooper pair" $\hat{b}(\mathbf{r}) = \sum_{\mathbf{k},\mathbf{q}} \chi_l(\phi) \hat{\psi}(\mathbf{k} + \mathbf{q}/2) \hat{\psi}(\mathbf{p} - \mathbf{q}/2) e^{i\mathbf{q}\cdot\mathbf{r}}$, with ϕ being the angle between the direction of the vector $\mathbf{k}$ and the x-axis and $\chi_l(\phi)$ is the function, which enforces the l-wave symmetry of the gap in the mean-field.

The first step is to integrate out the fermions and express the action in terms of the order parameter $\Delta_{\mathbf{k}} = \sum_{\mathbf{k},\mathbf{k}'} V(\mathbf{k}, \mathbf{k}') F(\mathbf{k}' - \mathbf{q}/2, \mathbf{k} + \mathbf{q}/2) e^{i\mathbf{q}\cdot\mathbf{r}}$, where according to Eq. (1) the interaction $V(\mathbf{k}, \mathbf{k}') = -\lambda_l \chi_l(\phi) \chi_l(\phi')$ and F is the standard Gor'kov's Green's function. In what follows we will concentrate on the spatial dependence of the gap and assume that its symmetry in the $\mathbf{k}$-space is preserved: $\Delta_{\mathbf{k}}(\mathbf{r}) = \Delta(\mathbf{r})\chi_l(\mathbf{k}) \equiv \Delta_0 f(\mathbf{r})\chi_l(\mathbf{k})$. Using these notations, we arrive at the following free energy for the system expressed in terms of the inhomogeneous order parameter $\Delta(\mathbf{r})$

$$\mathcal{F}[\Delta, U] = \frac{1}{2} \int_{1,2} \Delta^*(\mathbf{r}_1) A(\mathbf{r}_1, \mathbf{r}_2) \Delta(\mathbf{r}_2) \qquad (2)$$
$$+ \frac{1}{4} \int_{1,2,3,4} \Delta^*(\mathbf{r}_1) \Delta^*(\mathbf{r}_2) B(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \Delta(\mathbf{r}_3) \Delta(\mathbf{r}_4),$$

where $A(\mathbf{r}_1, \mathbf{r}_2) = \lambda_l^{-1} \delta(\mathbf{r}_1 - \mathbf{r}_2) - C(\mathbf{r}_1, \mathbf{r}_2)$ and the Cooperon, $\hat{C}$, is a random matrix expressed through the Green's functions (before averaging over disorder) as follows $C(\mathbf{r}_1, \mathbf{r}_2) =$
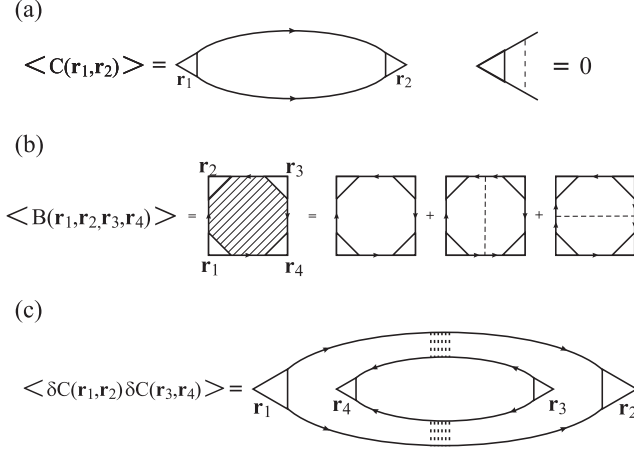
(a)

(b)

(c)

FIG. 1: (a) The particle-particle bubble for an anisotropic gap superconductor. This Cooperon diagram contributes to the coefficient in the quadratic term of the Ginzburg-Landau expansion (2). All disorder vertex corrections to the Cooperon vanish.; (b) Pictorial representation for the coefficient in quartic term of the Ginzburg-Landau expansion (2). (c) One of the UCF-type diagrams, which contribute to the mesoscopic fluctuations of the transition point and mesoscopic gap fluctuations.

$T \sum_{\varepsilon_n} G(\varepsilon_n; \mathbf{r}_1, \mathbf{r}_2) G(-\varepsilon_n; \mathbf{r}_1, \mathbf{r}_2)$. As long as we are interested in the location of the classical (finite-temperature) phase transition, the dynamics of the order parameter and the Cooperon are not important. We present the Cooperon as a superposition of a local "mean-field" part and a disorder dependent correction $\hat{C} = \langle \hat{C} \rangle + \delta\hat{C}$. The "mean-field" part is diagrammatically described by a simple Cooper bubble, without the disorder ladder (see Fig. 1a). Any disorder vertex correction vanishes due to the unusual symmetry of the gap. The line where the average Ginzburg-Landau coefficient vanishes $\langle A \rangle = 0$ determines the mean-field transition and leads to the well-known Abrikosov-Gor'kov's equation[15]

$$\ln \frac{T_{p0}}{T_p} = \psi\left(\frac{1}{2} + \frac{1}{4\pi T_p \tau}\right) - \psi\left(\frac{1}{2}\right), \qquad (3)$$

where T_{p0} is the pairing temperature without disorder and τ is the scattering time. This equation implies that the pair-breaking effect of the conventional disorder potential in an anisotropic gap superconductor [i.e., $\int d\phi \chi(\phi) = 0$] is identical to that of a time-reversal perturbation in an s-wave superconductor.[16] We reiterate that Eq. (3) is a result of the averaging over disorder in the sample. Below we study mesoscopic corrections to this result, which qualitatively can be interpreted as local changes in the scattering time $\tau(\mathbf{r})$ in Eq. (3).

We note that strictly speaking the nonlinear operator in the quartic term of Eq. (2) is also random, however its fluctuations can be neglected near the transition and its mean field value can be used $\langle B \rangle$. This coefficient is pictorially described by the Hikami-box diagram in Fig. 1b. A straightforward calculation of this diagram gives the following general result:

$$\langle B \rangle = -\frac{\nu}{16\pi^2 T^2}\left[\frac{\alpha}{12}\psi'''\left(\frac{1}{2} + \alpha\right) + \overline{|\chi_l|^4}\psi''\left(\frac{1}{2} + \alpha\right)\right], \quad (4)$$

where ν is the density of states, $\alpha = (2\pi T_p \tau)^{-1}$, and the overline implies averaging over the Fermi surface. In the clean limit, Eq. (4) reproduces the result of Feder and Kallin[17], $\langle B \rangle = 7\zeta(3)\nu/(8\pi^2 T^2)$. We note that the dirty limit is not reasonable in the context of an anisotropic gap superconductor, since it implies that the pairing temperature is suppressed to zero and there is no superconductivity. The maximum impurity concentration which allows for superconductivity (the quantum critical point) is $T_{p0}\tau_{\mathrm{QPT}} = \gamma/\pi \sim 1$, where $\gamma \approx 1.781$ is the exponential of the Euler's constant.

To find the local variations of the transition temperature, we consider the following eigenvalue problem for the random matrix $\delta\hat{C}$

$$\frac{1}{\nu}\int d^2\mathbf{r}' \delta C(\mathbf{r}, \mathbf{r}')\Delta(\mathbf{r}') = \epsilon\Delta(\mathbf{r}) \qquad (5)$$

and define the probability distribution function (PDF) of its eigenvalues $\rho(\varepsilon) = \left\langle \delta\left(\epsilon - \epsilon\left[\delta\hat{C}\right]\right)\right\rangle$. The averaging is performed over the PDF of the random Cooperon matrix, which we assume Gaussian $P\left[\delta\hat{C}\right] \propto \exp\left[-\frac{1}{2}\delta\hat{C} * \hat{K}^{-1} * \delta\hat{C}\right]$, where the asterisk implies a convolution over the two spatial variables and the operator $\hat{K}$ corresponds to the correlator of two Cooperon operators, which in position representation has the form $K(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3, \mathbf{r}_4) = \langle \delta C(\mathbf{r}_1, \mathbf{r}_2)\delta C(\mathbf{r}_3, \mathbf{r}_4)\rangle$. This disorder-averaged correlator can be calculated using the standard diagrammatic technique (see Fig. 1c). These diagrams are topologically equivalent to the universal conduction fluctuation (UCF) diagrams. However there are important differences: (i) First, here we are interested in the Cooper channel and (ii) Second, we are interested in the *local* physics, not in a long-wavelength behavior of the correlator.

Using the standard technique,[18] we find the following expression for the correlator

$$K_1[\{\mathbf{r}_i\}] = \delta(\mathbf{r}_1 - \mathbf{r}_4)\delta(\mathbf{r}_2 - \mathbf{r}_3)\left[\frac{2\tau\overline{|\chi_l|^2}}{4\pi^2 D\nu}\right]^2 \qquad (6)$$

$$\times \int\limits_{\tau \to 0}^{\infty}\int\limits_{\tau \to 0}^{\infty}\frac{dt_1 dt_2}{t_1 t_2 (t_1 + t_2)^2}\exp\left[-\frac{t_1 + t_2}{4Dt_1 t_2}|\mathbf{r}_1 - \mathbf{r}_3|^2\right].$$

Here the index "1" implies that we consider only one among all possible UCF-type diagrams. However, they all contribute equally to the correlator of interest and lead to a combinatorial factor of c, which is equal to $c = 12$ in the orthogonal ensemble and $c = 6$ in the unitary ensemble (e.g., in the presence of a magnetic field).

The PDF of the local transition temperatures and the corresponding gap amplitudes can be obtained using the optimal fluctuation method[19]

$$\langle\rho(\epsilon)\rangle \propto \exp\left[-\frac{1}{2}\frac{\epsilon^2}{\left\langle f \otimes f \left|\hat{K}\right| f \otimes f\right\rangle}\right], \qquad (7)$$

where the eigenvalue ϵ has the physical meaning of a local pairing temperature fluctuation and $f(\mathbf{r})$ is a normalized function, which describes the spatial profile and the shape of a single disorder-induced superconducting [if $\epsilon > (T - \langle T_p \rangle)/T$] or
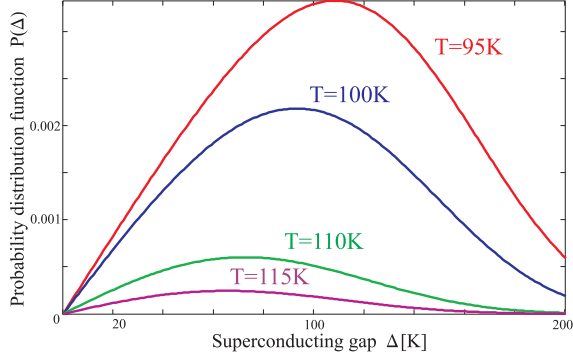
FIG. 2: (Color online) Plotted are distribution functions of the superconducting gap Δ for various temperatures, T. We assumed the following parameters: $g = 10$, $E_F/T_p \sim 40$, $T_p = 93\ K$. The latter choice is motivated by the experimental work[2] on $Bi_2Sr_2CaCu_2O_{8+\delta}$, where inhomogeneities have been observed. The temperature behavior of the PDF following from the mesoscopic fluctuation theory is qualitatively similar to that observed in experiment.[2] However, we note here that since our approach is based on the BCS theory, the theoretical results may not provide a quantitatively accurate description of the cuprates.

metallic [if $\epsilon < (T - \langle T_p \rangle)/T$] puddle. Strictly speaking the latter function must be found from a non-linear integral equation $\epsilon f(\mathbf{r}) = \Lambda \int_{1,2,3} f^*(\mathbf{r}_1) f^*(\mathbf{r}_2) K(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}) f(\mathbf{r}_3)$ (where Λ is a Lagrange multiplier which appears in the optimal fluctuation method; see Ref. [19] for technical details). However, one can get a quantitatively reliable description of the PDF by considering the puddle function to be a Gaussian of a characteristic size ξ, i.e., $f(r) = (\pi\xi^2)^{-1} \exp\left(-\frac{r^2}{2\xi^2}\right)$. In principle, one can study the distribution of puddle shapes by decomposing the function $f(\mathbf{r})$ into spherical harmonics. We do not attempt a study of the puddle shapes here, but just point out that "higher-orbital momentum puddles" are less probable than spherically symmetric ones; the probability of finding a droplet with "momentum" m scales as $p_m \propto p_0^m$, where p_0 is the probability of a spherical puddle. To find the latter we explicitly calculate the correlator in Eq. (7) and find $\left\langle f \otimes f \left| \hat{K} \right| f \otimes f \right\rangle = \frac{3c}{4\pi} \frac{l^2}{\xi^2} \frac{1}{g^2}$, where ξ is the size of a puddle, l is the mean free path, and $g = E_F\tau/\pi$ is the dimensionless conductance. This leads to the following PDF of T_p's (7):

$$\left\langle \rho(\xi, T_p) \right\rangle \propto \exp\left[-\frac{4\pi}{3c} \left(\frac{\xi}{l}\right)^2 g^2 \left(\frac{T_p - \langle T_p \rangle}{\langle T_p \rangle}\right)^2 \right]. \quad (8)$$

We note that by applying a magnetic field, one can cross over from the orthogonal to the unitary ensemble and change the combinatorial factor in (8) from $c = 12$ to $c = 6$, which may be experimentally testable and should manifest itself as a diminishing of the random $T_p(H)$ or gap distribution width exactly by the factor of two. To find the PDF of the superconducting gaps, we can just use the Ginzburg-Landau equation (2) and set $\Delta_0 = \sqrt{\nu(T_p - T)/(\langle T_p \rangle \langle B \rangle)}$, where $\langle B \rangle$ is given by Eq. (4). We note that it makes sense to consider a finite-size droplet with a well-defined local transition temperature or a gap, only if the size of the droplet is much larger than the
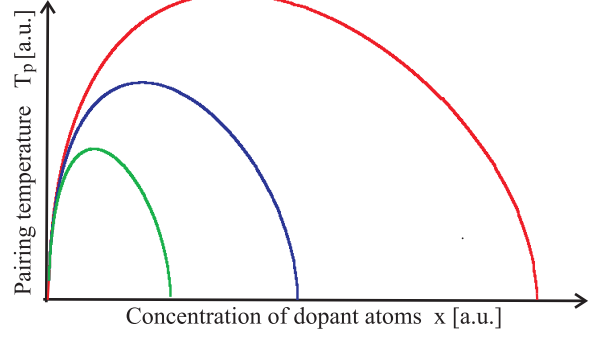

FIG. 3: (Color online) This figure is to illustrate the qualitative discussion in the text about the possibility of a dome-shaped doping dependence of the pairing temperature within the Abrokosov-Gor'kov theory. The figure shows a typical dependence of T_p on "doping," x, in the toy model of the Abrikosov-Gor'kov theory with both $T_{p0}(x)$ and $\tau(x)$ being dependent on the same doping parameter. The graphs are solutions of the following equation for t_p: $\ln\left(x^{1-\alpha}/t_p\right) = \psi\left(1/2 + x\delta/t_p\right) - \psi(1/2)$, where t_p is a dimensionless pairing temperature and δ characterizes the strength of disorder. The graphs correspond to $\alpha = 1/2$ and $\delta = 0.1$ (red line), 0.15 (blue line), and 0.2 (green line).

coherence length $\xi \gg \xi_{\Delta_0}$. The opposite limit corresponds to the case of a mesoscopic superconducting nanograin in which the notion of the gap is not well-defined (see Ref.[20] for a review). Therefore, the smallest possible size of the puddle with a well defined gap Δ_0 is $\xi_{\min} \sim v_F/\Delta_0$, which implies that the dimensionless ratio $\xi_{\min}/l \sim (T_p\tau)^{-1}$. The latter parameter is of order one and thus the PDF of the gaps takes the form (see also Fig. 2)

$$P[\Delta] \sim \frac{2g\sqrt{\beta}b\Delta}{\sqrt{\pi}\langle T_p \rangle^2} \exp\left[-\beta g^2 \left(\frac{b\Delta^2}{\langle T_p \rangle^2} + \frac{T - \langle T_p \rangle}{\langle T_p \rangle} \right)^2 \right], \quad (9)$$

where in the d-wave case, $b \sim -\left[\frac{\alpha}{3}\psi'''\left(\frac{1}{2} + \alpha\right) + 3\psi''\left(\frac{1}{2} + \alpha\right)\right]/(32\pi^2)$, $\alpha = (E_F/\langle T_p \rangle)(2\pi^2 g)^{-1}$, and $\beta \sim 1$. Note that in Eq. (9) we have omitted a term proportional to $\propto \delta(\Delta)$, which describes normal regions. The physical picture, which emerges for such a disordered superconductor is that right above the mean-field transition temperature there exist rare superconducting islands separated by "normal regions" (which are still very close to the local transition temperature). We note that since the parameter gT_{p0}/E_F is at best of order one, the "mesoscopic Griffiths phase" overlaps with the Ginzburg region of strong superconducting fluctuations.[21] This leads to the conclusion that the Griffiths phase is a mixture of superconducting islands and metallic regions with strongly suppressed density of states.

Even though our quantitative description directly applies only to a weakly coupled BCS superconductor, we believe that some aspects of the theory are relevant to the cuprates as well (the importance of disorder effects for the cuprates have been discussed previously, see, e.g. [22,23,24,25,26,27]). But

first, we make the following curious observation: In a high-T_c superconductor the major source of disorder is presumably the dopant atoms. But this access oxygen is also the source of the carriers, which lead to superconductivity in the first place. Thus, the "clean" pairing temperature in the Abrikosov-Gor'kov's formula (3) should explicitly depend on the doping level, $T_{p0}(x)$ (e.g. through a BCS-like high-energy cut-off or possibly via a locally modulated electron pairing interaction.[25]) The scattering time depends on x too and contributes to superconductivity suppression. Thus there are two competing effects of the dopants: They both enhance and suppress superconducting properties. An interesting result, which follows from the simple Abrikosov-Gor'kov's equation (3), is that even if $T_{p0}(x)$ is monotonically increasing with x, but $T_{p0}(x)\tau(x) \propto x^{-\alpha}$ with $\alpha > 0$, then the actual pairing temperature doping dependence has a dome-shaped form. An example of such a dependence for $\alpha = 1/2$ is plotted in Fig. 3.

The mesoscopic disorder fluctuations in the Abrikosov-Gor'kov theory may not correspond to the modulations of the transition temperature in a high-T_c superconductor, but should be related to the modulations of the pseudogap temperature, T_*, which is believed to be the onset of Cooper pairing (i.e., to T_p, but not T_c). The "pseudogap" region $T_c < T < T_p$ presumably represents the regime of strong phase fluctuations[28] and the superconducting transition is expected to be that of XY-type. In the latter scenario, the transition temperature is proportional to the superfluid density, which in turn is directly related to the local value of the gap. The mean field gap is determined by the deviation $(T_p - T)$ from the local pairing temperature (e.g., via a non-linear BCS-like self-consistency equation $\delta S/\delta \Delta = 0$). If $T_p(\mathbf{r})$ fluctuates due to disorder, so does $\Delta(\mathbf{r})$ and, quite generally, the local gap should "follow" local T_p. In fact, such a correlation has been observed in experiment.[2] In the model of uncorrelated short range disorder, the only possible result for the corresponding PDF is Eq. (9), with Δ centered in the vicinity of the mean-field gap at the given temperature $P[\Delta] \propto \exp\left\{-\beta g^2 \left[1 - \Delta/\langle\Delta(T)\rangle\right]^2\right\}$. In a clean material, the corresponding regime of mesoscopic fluctuations is very narrow unless there are other types of disorder effects, such as "structural disorder" (e.g., extended defects, warping of the 2D planes, etc.), which can be modelled as a random diffusion coefficient[13] $\langle D(\mathbf{r})D(\mathbf{0})\rangle = \langle D\rangle^2 a^2 \delta(\mathbf{r})$. These phenomena lead to qualitatively the same effect of random T_p and Δ, but, may occur at the length-scales much larger than the mean-free path and become important in a much wider range of parameters [the width of the distribution is determined by $(a/l)g^{-1}$ instead of g^{-1}]. We also note that if indeed the local gap fluctuations observed in experiment[2] are related to mesoscopic disorder effects, they should be correlated with the pinning properties,[29] i.e., the width of the gap distribution should be proportional to the critical current j_c in the collective pinning regime[13], $\left\langle (1 - \Delta/\langle\Delta\rangle)^2\right\rangle \propto (j_c/j_{c0})(H/H_{c2})$ (where j_{c0} is the critical current in zero field).

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